Extending Hölder-McCarty inequality to accretive normal matrices/operators

Morteza Seddighin

Indiana University East, Richmond, Indiana, USA

Abstract

The concept of Antieigenvalue for a positive operator was first introduced thirty years ago by Karl Gustafson. Antieigenvalues have applicatoins in matrix statistics. Some of these applications were discussed by Karl Gustafson and the author last year in IWMS07, Windsor, Ontario, Canada. In this paper we define the concept of a q-f antieigenvalue and use convex optimization techniques to generalize Hölder-McCarthy inequality. Let A be a positive operator on a Hilbert space H satisfying $M \geq A \geq m > 0$. Also let $f(t)$ be a real valued convex function on $[m, M]$ and q be a real number, then the inequality

$$(f(A)x, x) \leq \frac{(mf(M) - Mf(m))}{(q-1)(M-m)} \left(\frac{(q-1)(f(M) - f(m))}{q(mf(M) - Mf(m))} \right)^q (Ax, x)^q,$$

which holds for every vector x, under certain restrictions on f and q , is called the Hölder-McCarthy inequality. We will generalize the Hölder-McCarthy inequality from positive operators to accretive normal operators.

Keywords

q-f antieigenvalue, Hölder-McCarty inequality.